Non-locality requires fine tuning and multi-degenerate vacua

D.L. Bennett[1] and H.B. Nielsen[2]

(1) Brookes Institute for Advanced Studies, Bøgevej 6, 2900 Hellerup, Denmark
(2) The Niels Bohr Institute, Blegdamsvej 17, 2100 Copenhagen Ø, Denmark

Abstract

Our Multiple Point Principle (MPP) states that the realized values for e.g. the parameters of the standard model correspond to having a maximally degenerate vacuum. In the original appearence of MPP the gauge coupling values were predicted to within experimental uncertainties. A mechanism for fine-tuning follows in a natural way from the MPP. Using the cosmological constant as a example, we attempt to justify the assertion that at least a mild form of non-locality is inherent to fine-tuning. This mild form - namely an interaction between pairs of spacetime points that is identical for all pairs regardless of spacetime separation - is insured by requiring non-local action contributions to be reparametrization invariant. However, even this form of non-locality potentially harbours time-machine-like paradoxes. These are seemingly avoided by the MPP fine-tuning mechanism. A (favorable)comparison of the results of MPP in the original lattice gauge theory context with a new implementation with monopoles that uses MPP at the transition to a monopole condensate phase is also described.

1. Introduction

The Multiple Point Principle (MPP) states that fundamental physical parameters assume values that correspond to having a maximal number of different coexisting "phases" in the vacuum. There is phenomenological evidence suggesting that some or all of the about 20 parameters in the Standard Model (SM) that are not predicted within the framework of the SM correspond to the MPP values of these parameters[1, 2, 3]. The MPP also provides a natural way to understand how fine-tuning comes about in a manner analogous to the way that the coexistence of the ice and liquid phases of water is required for a finite range of energies for the system all of which result in the fine-tuning of the temperature to 0^oC (at 1 atm.).

A cursory examination of the relation between the bare and dressed cosmological constant that is maintained dynamically seems to demand at least a mild form of non-locality which in turn makes us vulnerable to "matricide-like" paradoxes. Postulating MPP as a law of Nature seems to avoid paradoxes and at the same time provides an eloquent mechanism of fine-tuning standard model parameters to experimentally observed values.

In this contribution, we outline a new way of applying MPP by arguing that new physics in the form of a fundamental (ontological) lattice (and the accompanying monopoles) would necessarily appear at a scale for which an infinite monopole self coupling gives an upper limit (by the Triviality Theorem). The monopoles inherent to this lattice undergo a phase transition to and from a monopole condensate phase. By arguing that letting λ run to infinity would set the (upper limit for the) fundamental cutoff scale and using that the gauge couplings approach constant values for "large" enough values of the self-coupling λ, we get predictions for the gauge couplings by letting a curve that shares a point with a monopole transition curve at small (negative) λ values run to infinite λ. These predictions are compared with the original lattice gauge theory implementation of MPP.

2. Non-locality seems inherent to the problem of fine tuning

In essence, solving the problem of fine tuning means finding a way to render couplings (or other intensive quantities) dynamical. This invariably leads to non-locality - at least in a mild sense - in a way exemplified by the fine tuning problem for the dressed cosmological constant [4]. If a coupling, e.g., the cosmological constant, is dynamical, locality dictates that the value of the coupling at some spacetime point can directly depend on that spacetime point and indirectly on other spacetime points at earlier times but certainly not on the future.

But if the bare cosmological constant immediately following the big bang is to already have its value fine tuned very exactly to the value that makes the dressed cosmological constant as small as that suggested phenomenologically, we have a problem with locality in the following sense: in order that the bare cosmological constant be relateable to the value of the dressed cosmological constant, the details of the dressed cosmological constant that will evolve in the future must be known at the time of big bang. So a strict principle of locality is not allowed if we want to have a dynamically maintained bare coupling and renormalization group corrections of a quantum field theory with a well-defined vacuum.

An allowable resolution would be to allow a mild class of non locality consisting of an interaction that is the same between any pair of points in spacetime independent of the distance between these points. It would be difficult to see that such an interaction is non-local. Instead, such spacetime omnipresent fields - a sort of background that is forever everywhere the same - would likely be interpreted simply as constants of Nature.

The reparametrization invariance of general relativity implies this symmetry (i.e., the same interaction between any pair of spacetime points). Leting $\phi(x)$ stand for all the fields (and derivatives of same) of the theory, we use the fact that integrals (extensive quantities) of the form $I_{f_j}[\phi(x)] \overset{def}{=} \int dx^4 \sqrt{g(x)} f_j(\phi(x))$ as well as any function of such integrals are reparametrization invariant. Here the $f_j(\phi)$ are typically Lagrange densities. We get a reparametrization invariant but non-local action $\hat{S}_{nl}$ by taking a non-linear function of the funtionals $I_{f_j}[\phi(x)]$ as $\hat{S}_{nl}$.

Each extensive quantity $I_{f_j}[\phi(x)]$ has a fixed value - call it $I_{f_j\ fixed}$ (fixed in the sense of being a Law of Nature) for each imaginable Feynman path integral history of the Universe as it evolves from Big Bang at time t_{BB} to Big Crunch at time t_{BC}. The set $\{I_{fixed\ f_j}\}$ of allowed values for the I_{f_j} can be implemented as a δ-function in the functional integration measure that defines our reparametrization invariant non-linear (and therefore non-local) action $\hat{S}_{nl}$: $\exp(S_{nl}(\{I_{f_j}\})) = \prod_j \delta(I_{f_j}[\phi] - I_{fixed\ f_j})$.

3. The Multiple Point Principle (MPP)

The first appearence[1] of the MPP was in connection with predictions of the values of the non-Abelian gauge couplings. This was done in the the context of lattice gauge theory using our so-called family replicated gauge group[5, 6] G_{FRGG} (also sometimes referred to as $G_{Anti-GUT}$) which consists of the 3-fold replication of the Standard Model Group (SMG): $G_{FRGG} = SMG \otimes SMG \otimes SMG \overset{def}{=} SMG^3$ (in the extended version: $(SMG \times U(1))^3$) having one SMG factor for each generation of fermions and gauge bosons. We postulate that G_{FRGG} is broken to the diagonal subgroup (i.e., the usual SMG) at roughly the Planck scale.

In the original context of predicting the standard model gauge couplings, MPP asserts that the Planck scale values of the standard model gauge group couplings coincide with the multiple point, i.e., the point or (hyper)surface that lies in the boundary separating the maximum number of phases in the action parameter space corresponding to the gauge group G_{FRGG}. The (Planck scale) predictions for the gauge couplings are subsequently identified with the parameter values at the point in the action parameter space for the diagonal subgroup of G_{FRGG} that is inherited from the multiple point for G_{FRGG} after the Planck scale breakdown of the latter.

The phases to which we refer are usually dismissed as lattice artifacts (e.g., a Higgsed phase, a confined or Coulomb-like phase). Such phases have been studied extensively in the literature for gauge groups such as $U(1)$, $SU(2)$ and $SU(3)$). One typically finds first order phase transitions between confined and Coulomb-like phases at critical values of the action parameters.

We suggest that these lattices phases correspond to phases that may be inherent to any regulator. As a regulator in some form (be it a lattice, strings or whatever) is needed for the consistency of any quantum field theory, it is consistent to assume the existence of a fundamental regulator. The "artifact" phases that arise in a theory with such a regulator (that we have initially chosen to implement as a fundamental lattice) are accordngly taken as ontological phases that have physical significance at the scale of the fundamental regulator (e.g., lattice). The assumption of an ontological fundamental regulator implies the existence of monopoles in terms of which the regulator induced phase can also be studied[6].

Finding the multiple point in an action parameter space corresponding to the gauge group G_{RGG} is more complicated than for groups such as $U(1)$, $SU(2)$ or $SU(3)$ say. The boundaries between phases in the action parameter space (i.e., the phase diagram) must be sought in a high dimensional parameter space essentially because $G_{Anti-GUT}$ being a non-simple group has many subgroups and invariant subgroups.

In fact there is a distinct phase for each subgroup pair (K, H) where K is a subgroup and H is an invariant subgroup such that $H \lhd K \subseteq G_{FRGG}$. An element $U \in G_{FRGG}$ can be parameterized as $U = U(g, k, h)$ where the Higgsed (gauge) degrees of freedom are elements g of the homogeneous space G_{FRGG}/K. The (un-Higgsed) Coulomb-like and confined degrees of freedom are respectively the elements k of the factor group K/H and the elements $h \in H$.

4. The History of the Universe as a Fine-tuner

MPP functions as a fine-tuning mechanism when the extensive quantities $I_{fixed\ f_j}$ introduced in Section 2 happen to have values that can only be realized in a universe having two (or more) coexisting phases the transition between which is first order in which case the intensive quantity (typically a coupling) conjugate to $I_{fixed\ f_j}$ is fine-tuned. It may be useful to consider a familiar analogy: for a system consisting of H_2O there is a whole range of energies (corresponding to the heat of melting) for which the system is forced to be realized as coexisting ice and liquid phases in which case the energy-conjugate intensive parameter i.e., temperature, is fine-tuned (e.g., to 0^oC for a presure of 1 atm.). A better implementation of MPP would be the triple point of water where for a finite range of energies and volumes three phases meet and 3-1=2 intensive parameters (the temprature and presure) are fine-tuned to the triple point values.

In 4-space, one generic possibility for having coexistent phases would be to have a phase with ϕ_{us} in an early epoch including say the universe as we know it and a phase with ϕ_{other} in a later epoch:

$$I_{fixed\ f_j} = f_j(\phi_{us})(t_{ignit}-t_{BB})V_3 + f_j(\phi_{other})(t_{BC}-t_{ignit})V_3 \quad (V_3 \text{ is the 3-volume of the universe}) \tag{1}$$

where t_{ignit} is the "ignition" time (in the future) at which there is a first order phase transition from the vacuum at ϕ_{us} to the later vacuum at ϕ_{other}. The value of the "coupling constant" conjugate to $I_{fixed\ f_j}$ gets fine tuned (unavoidably by assumption of the coexistence of the two phases separated by a first order transition) by a mechanism that also depends on a phase that will first be realized in the future (at t_{ignit}). Such a mechanism is non-local. Note in particular that the right hand side of Eqn. 1 depends on t_{ignit}.

We want to formally define a "coupling constant" conjugate to some extensive quantity $I_{fixed\ f_j}$. Restrict the non-local action $\hat{S}_{nl} = \hat{S}_{nl}(\{I_j[\phi(x)]\})$ to being a non-local potential V_{nl} that is a function of (not necessarily independent) functionals: $V_{nl} \stackrel{def}{=} V_{nl}(I_{f_i}[\phi], I_{f_j}[\phi], \cdots)$. Define an effective potential V_{eff} such that

$$\frac{\partial V_{eff}(\phi(x))}{\partial \phi(x)} \stackrel{def}{=} \frac{\delta V_{nl}(\{I_{f_j}[\phi]\})}{\delta \phi(x)}\Big|_{near\ min.} = \sum_i \left(\frac{\partial V_{nl}(\{I_{f_j}\})}{\partial I_{f_i}} \frac{\delta I_{f_i}[\phi]}{\delta \phi(x)} \right)\Big|_{near\ min.} \tag{2}$$

$$= \sum_i \frac{\partial V_{nl}(\{I_{f_j}\})}{\partial I_{f_i}}\Big|_{near\ min.} f_i'(\phi(x))$$

The subscript "near min" denotes the approximate ground state of the whole universe, up to deviations of $\phi(x)$ from its vacuum value (or vacuum values for a multi-phase vacuum) by any amount in relatively small spacetime regions. The solution to Eq. (2) is

$$V_{eff}(\phi) = \sum_i \frac{\partial V_{nl}(\{I_{f_j}\})}{\partial I_{f_i}} f_i(\phi) \tag{3}$$

We identify $\frac{\partial V_{nl}(\{I_{f_j}\})}{\partial I_{f_i}}$ as intensive quantities conjugate to the I_{f_i}.

Consider now the effective potential (3) in the special case that $V_{nl}(\{I_{f_j}\}) = V_{nl}(I_2, I_4) \stackrel{def}{=} V_{nl}(\int d^4x \sqrt{g(x)}\phi^2(x), \int d^4y \sqrt{g(y)}\phi^4(y))$ in which case, (3) becomes

$$V_{eff} = \frac{\partial V_{nl}(I_2, I_4)}{\partial I_2}\phi^2(x) + \frac{\partial V_{nl}(I_2, I_4)}{\partial I_4}\phi^4(x) \stackrel{def}{=} \frac{1}{2}m_{Higgs}^2\phi^2(x) + \frac{1}{4}\lambda\phi^4(x) \tag{4}$$
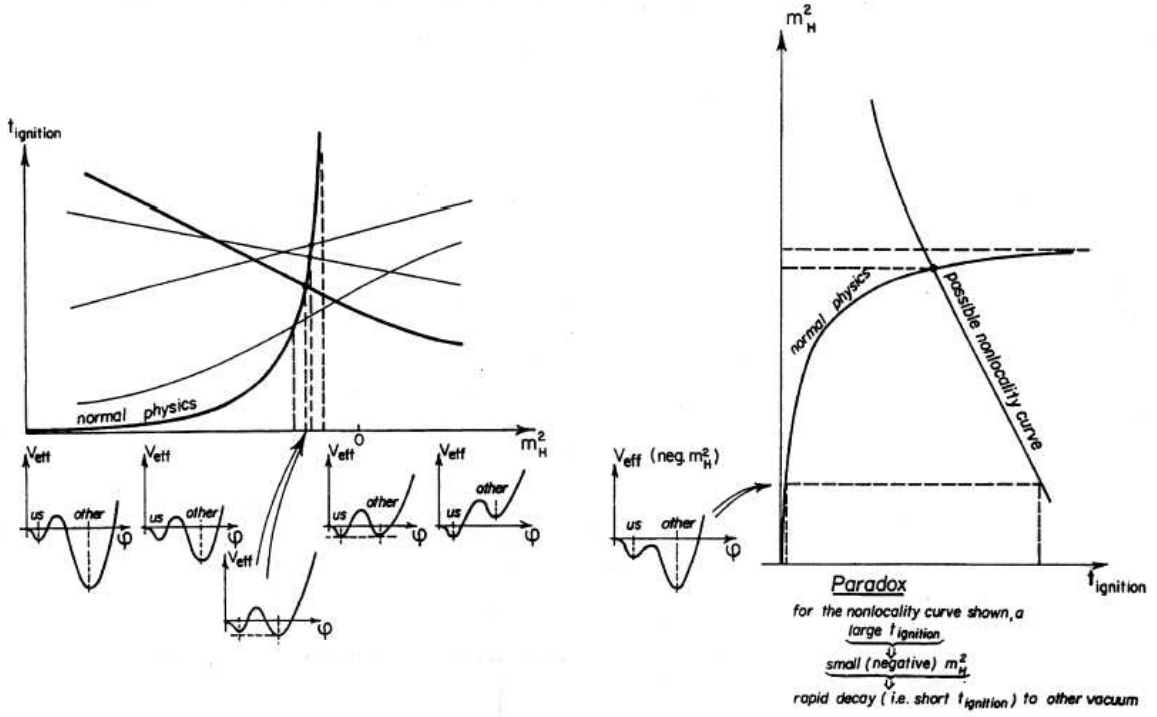
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Figure 1: (Left) The development of the double well potential and m_{Higgs} as a function of t_{ignit}. Note that all the more or less randomly drawn non-locality curves intersect the "normal physics" curve near where the vacua are degenerate (i.e., the MPP solution).

Figure 2: (Right) Many non-locality curves could lead to paradoxes similar to the "matricide" paradox. Such paradoxes are avoided if the value of m_{Higgs} is fine-tuned to the multiple point critical value. This corresponds to the intersection of the "normal physics" curve with the "possible nonlocality" curve.

where the right hand side of this equation, which also defines the (intensive) couplings m_{Higgs}^2 and λ, is recognised as a prototype scalar potential at the tree level. Of course the form of V_{nl} is, at least a priori, completely unknown to us, so - for example - the coupling constant m_{Higgs}^2 cannot be calculated from Eqn. 4. The potential of Eqn. 4 with $m_{Higgs}^2 < 0$ has an asymmetric minimun at, say, the value ϕ_{us} resulting in spontaneous symmetry breakdown in the familiar way.

Actually we want to consider the potential V_{eff} having the two relative minima ϕ_{us} and ϕ_{other} - both at nonvanishing values of ϕ - alluded to at the beginning of this section. The second minimum comes about at a value $\phi_{other} > \phi_{us}$ when radiative corrections to (4) are taken into account and the top quark mass is not too large[3, 7, 8]. Which of these vacua - the one at ϕ_{us} or ϕ_{other} - would be the stable one in this two-minima Standard Model effective Higgs field potential depends on the value of m_{Higgs}^2. Since I_2 and I_4 are functions of t_{ignit} (as seen from Eqn. 1 with $f_j = \phi^2$ or ϕ^4), $m_{Higgs}^2 \stackrel{def}{=} \frac{\partial V_{nl}(\{I_2,I_4\})}{\partial I_2}$ is also a function of t_{ignit}.

Let us first use "normal physics" to see how the relative depths of the two minima of the double well are related to m_{Higgs}^2 and to t_{ignit}. It can be deduced from[8] that a large negative value of m_{Higgs}^2 corresponds to the relative minimum $V_{eff}(\phi_{other})$ being deeper than $V_{eff}(\phi_{us})$ (in which by assumption the Universe starts off following Big Bang) than for less negative values of m_{Higgs}^2 (see Fig. 1). It can also be argued quite plausibly that a minimum in V_{eff} at ϕ_{other} much deeper than that at ϕ_{us} would correspond to an early (small) t_{ignit} inasmuch as the "false" vacuum at ϕ_{us} would be very unstable. However,

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as the value of the potential at ϕ_{other} approaches that at ϕ_{us}, t_{ignit} becomes longer and longer and approaches infinity as the values of V_{eff} at ϕ_{us} and ϕ_{other} become the same. The development of the double well potential and m^2_{Higgs} as a function of t_{ignit} is illustrated in Fig. 1. Note that the larger the difference $|\phi_{other} - \phi_{us}|$ the more the realization of say $I_{fixed\,2}$ will in general depend on t_{ignit}. If $\phi_{us} = \phi_{other}$, t_{ignit} plays no role in realizing e.g. $I_{fixed\,2}$ and the value of m^2_{Higgs} becomes independent of t_{ignit}.

5. Avoiding paradoxes arising from non-locality

In general the presence of non-locality leads to paradoxes. While the form that the non-local action (or potential V_{nl} in this discussion) is unknown to us, we make the 4 generically representative guesses portrayed as the 4 non-locality curves in Fig. 1. In particular, non-locality curves having a negative slope as a function of t_{ignit} lead to paradoxes in the following manner. Consider the non-locality curve in Fig. 1 drawn with bold line that is redrawn in a rotated position in Fig. 2. Let us make the assumption that t_{ignit} is large and see that this leads to a contradiction. Assuming that t_{ignit} is large, it is seen from the non-locality function in Fig. 2 (call it $m^2_{Higgs\,nl}(t_{ignit})$ to distinguish it from the "normal physics" $m^2_{Higgs}(t_{ignit})$) that this implies that the "normal physics" m^2_{Higgs} has a large negative value. But a large negative value of m^2_{Higgs} corresponds in "normal physics" to a (false) vacuum at ϕ_{us} that is very unstable and therefore to a very short t_{ignit} corresponding to a rapid decay to the stable vacuum at ϕ_{other}. So the paradox appears: the assumption of a large t_{ignit} implies a small t_{ignit}. This happens because in general $m^2_{Higgs}(t_{ignit}) \neq m^2_{Higgs\,nl}(t_{ignit})$ and is akin to the "matricide" paradox encountered for example when dealing with "time machines". It has been suggested [9, 10, 11] that Nature avoids such paradoxes by choosing a very clever solution in situations where these paradoxes lure.

In the case of the paradoxes that can come about due to non-locality of the type considered here, a clever solution that avoids paradoxes is available to Nature in the form of the Multiple Point Principle (MPP). The MPP solution corresponds to the intersection of the "normal physics" curve and the "non-locality curve" in Fig. 2. because here the vacua at ϕ_{us} and ϕ_{other} are (essentially) degenerate. But at this intersection point, $m^2_{Higgs}(t_{ignit}) = m^2_{Higgs\,nl}(t_{ignit})$ so the paradox is avoided. So the paradox is avoided at the multiple point. But at the multiple point, an intensive parameter has its value fine-tuned for a wide range of values of the conjugate extensive quantity. Fine-tuning can therefore be understood as a consequence of Nature's way of avoiding paradoxes that can come about due to non-locality.

6. Determining MPP gauge couplings at transition to monopole condensate

Until now we have talked about the determination of gauge couplings by finding the point (or surface) - the multiple point (surface) - where the maximum number of "lattice artifact" phases come together in the action parameter space of a lattice gauge theory. Here we briefly sketch an alternative that potentially replaces the assumption of an ontological lattice by instead producing the different phases using the assumption of monopoles. Monopoles can cause different phases by condensing or not. Then the MPP prediction is that the couplings realized in Nature are the values at the transition between a Coulomb-like phase and a monopole condensate. Because it is not important whether the monopoles

are lattice artifact monopoles or fundamental monopoles or whatever, this way of determining the MPP gauge couplings offer the possibility of exonerating the original method that considers "lattice artifact" phases.

A simple effective dynamical description of confinement in a pure $U(1)$ lattice gauge theory is the dual Abelian Higgs model of scalar monopoles[12] We shall shortly see that the dualized $U(1)$ lattice theory can be rewritten as an **R** lattice gauge theory on the dual lattice with a "non-linear" Higgs field. In earlier work[13] the formulation dual to the renormalization group improved Coleman-Weinberg effective potential (at tree level this is just Eqn. 4) was considered in the two loop approximation. In this work the transition to a monopole condensate was found along a phase transition curve in (λ, g^2) space situated in the region with negative λ of the order of -10. Here λ and g^2 are running couplings for which one would think that the renormalization point μ should be taken to be of the order of the Higgs monopole mass or the VEV of the monopole field in the condensed phase. The negative λ is not alarming for the existence of the phase transition even if one requires a bottom for the Hamiltonian because by running to a higher renormalization point (identified with field strength) the self-coupling can run positive. In the earlier work[13] the MPP philosophy was to take the actual coupling directly related to the couplings on the phase transition curve just mentioned. In the present paper we want to consider MPP applied to bare couplings. If bare couplings are considered, one must declare the scale of the bare coupling. To avoid a negative λ, one should at least choose the ontological cutoff scale high enough that λ is positive. On the other hand we know from the Triviality Theorem that new physics (e.g., a fundamental lattice) must show up at the latest at the energy where the monopole self-coupling λ becomes infinite due to renormalization group running.

So we have an allowed interval for the scale of the fundamental lattice: it must be greater than that for which $\lambda_{running}$ becomes positive but less than the scale at which λ runs to infinity.

Now it turns out that as λ runs to "large" values, the dependence of g^2 on λ becomes very weak. So by running the $g^2(\lambda)$ curve from the one point that it has in common with the above-mentioned phase transition curve along which there is a Coulomb - condensate phase transition to "large" enough λ values, we get a good candidate for the running g^2 value at the scale of the fundamental regulator (lattice).

So what we now need is a justification for assuming that λ is "large" enough. There are a couple of justification candidates. One approach would be to make the assumption that the approximation of a continuum spacetime is good all the way up to the regularization scale given by the Triviality Theorem bound. Since this bound is determined by $\lambda \overset{running}{\to} \infty$ we have justified the assumption of large λ. The second approach to justifying the assumption of large λ entails the assumption of an ontological lattice and a little calculation.

The calculation would start by using dualization to rewrite a $U(1)$ lattice gauge theory into a theory with the gauge group **Z** (under addition) on the dual lattice links $\overset{dual}{\frown}$. Next we endeavor to get this integer gauge group from the group of real numbers **R** by using a weighting which for each dual link $\overset{dual}{\frown}$ is a sum of infinitely many δ-functions in the gauge group **R** minus an integer n over which we sum:

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$$\prod_{dual}\sum_{n\in\mathbf{Z}}\delta(\theta_{\mathbf{R}}(\overset{dual}{\rule{1.2em}{0.8pt}})-2\pi n).$$

By putting in the δ-function factor, we essentially go from a real number gauge group to an integer gauge group. Now the trick is to get this series of δ-function weightings for each dual link by formally introducing a series of "non-linear" Higgs fields i.e., a scaler field on each site that has the complex unit circle as its target space. By choosing the unitary gauge we can take the value unity for this non-linear gauge field. Then the lowest order action contribution (from the kinetic term of the Higgs field) will be $\beta_{\mathbf{R}}cos(\theta_{\mathbf{R}}(\overset{dual}{\rule{1.2em}{0.8pt}}))$ for each dual link (summed over all dual links). If $\beta_{\mathbf{R}}$ goes to infinity, the exponentiated action for each link will be a weight of the form we want (i.e., a sum of infinitely many δ-functions).

7. Conclusion

We attempt to justify the assertion that fine-tuning in Nature seems to imply a fundamental form of non-local interaction. This could be manifested in a phenomenologically acceptable form as everywhere in spacetime identical interactions between any pair of spacetime points. This would be implemented by requiring the non-local action to be diffeomorphism invariant.

Next we put forth our multiple point principle[1, 2] which states that coupling parameters in the Standard Model tend to assume values that correspond to the values of action parameters lying at the junction of a maximum number of regulator induced phases (e.g., so-called "lattice artifact phases") separated from one another in action parameter space by first order transitions. The action. which of course is defined on a gauge group (e.g., the non-simple SM gauge group) governs fluctuation patterns along the various subgroup combinations (K, H) with $H \lhd K \subseteq G$ that characterize the phases that come together at the multiple point.

We then consider extensive quantities that are functions of functionals $I_{f_j}[\phi(x)]$ that are essentially Feymann path histories of the Universe for functions $f_j(\phi)$ of the fields $\phi(x)$ and derivatives of these fields. We then think of the generic situation in which these extensive quantities can happen to be fixed at values that require the universe to be realized as two or more coexisting phases[4]. We draw on the analogy to the forced coexistence of ice and liquid water that occurs for a whole range of possible total energies because of the finite heat of melting (first order phase transition). With our multiple point principle, the intensive quantities (couplings) conjugate to extensive quantities fixed in this way become fine-tuned in a manner analogous to the fine tuning of temperature to $0^o C$ (at 1 atm.) when the total energy of a system of H_2O is such that the system can only be realized as coexisting ice and liquid phases.

One generic way of having coexisting phases in a quantum field theory in 3+1 dimensions would be to have different phases in different epochs of the lifetime of the Universe with phase transitions occuring at various times in the course of the lifetime of the Universe. If the transitions were first order, one would have fine-tuning of (intensive) couplings conjugate to extensive quantitity values that can only be realized by having coexisting (i.e., more than one) phases. But such a fine-tuning would involve non-locality:

Table 1: Table of values of $\alpha^{-1}(\mu_{Pl.})$. Recall that these values are those for each of the three $U(1)$s, each of the three $SU(2)$s and each of the three $SU(3)$s in our family replicated gauge group G_{FRGG} because G_{FRGG} is the 3-fold Cartesian product of the usual standard model group (SMG). Following the Planck scale breakdown of G_{FRGG} to the diagonal subroup (isomorphic to the usual SMG) the $\alpha^{-1}(\mu_{Pl.})$ values for the non-Abelian subgroups get multiplied by a factor 3 whereas $\alpha_{U(1)}^{-1}(\mu_{Pl.})$ gets enhanced by a factor somewhat greater than six (≈ 6.5) for reasons having to do with the three $U(1)$s of G_{FRGG} being Abelian.[2] For comparison, the last row gives experimental values of $\alpha^{-1}(\mu_{Pl})$ that have been extrapolated to Planck scale using the renormalization group (with minimal standard model) and subseqently divided by factors 3 and 6.5 in respectively the non-Abelian and Abelian cases.

	U(1)	SU(2)/Z_2	SU(3)/Z_3
Naive continuum limit	12.4	21.7	26.7
Parisi Improved	8.25	15.4	16.3
Monopole 1-loop	7.20	15.0	18.2
Experimental values (Planck scale)[1]	8.22	16.4	17.9

the fine-tuned values of coupling constants would depend on future phase transitions into phases that do not even exist at the time such couplings are fine-tuned.

Even non-locality of this sort (i.e., non-localy manifested as a diffeomorphism invariant contribution to the action) can lead to paradoxes of the "matricide paradox" type. We argue that such paradoxes are avoided when Nature chooses the multiple point principle solution to the problem of finetuning[4].

The first formulation of the MPP arose in our predictions of the SM gauge coupling constants[1, 2]. This was done in the context of lattice gauge theory using our family replicated gauge group $SMG \otimes SMG \otimes SMG$ having one SMG for each family of fermions (and bosons!)[5, 6]. This is broken to the diagonal subgroup (which is isomorphic to the usual SMG) at roughly the Planck scale. The predictions for the "Naive continuum limit" and "Parisi Improved" values of $\alpha^{-1}(\mu_{Pl})$ (see table) rely on on the assumption that what are usually regarded a "lattice artifact" are in fact ontological. This assumption is avoided in recent work that uses a monopole technology to make MPP predictions of gauge couplings. These is in rather good agreement of the "Monopole 1-loop" with the original predictions (see table of $\alpha^{-1}(\mu_{Pl})$ values)

We use the Triviality Theorem in support of our requirement that the monopole (on the dual lattice) attain an infinite self-coupling at the lattice scale. This requirement excludes having a fixed point in the running λ value before reaching the lattice scale because $\beta_\lambda = 0$ at any scale under that of the lattice would stop the running before λ becomes infinite.

In the two loop approximation it turns out that zeros in β_λ show up at $g^2 \approx 19$ or above. But if we trust the Triviality Theorem, we must assume that going to three loops (or more) would remove the two loop (artifact) β_λ zeros or that we already are at sufficiently strong g^2, λ so that the optimal order is one-loop and that going to two-loops introduces error. For now, we use the one loop calculation in calculating the values in the table and postpone a two loop or other attempts at a more accurate treatment.

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